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Estimation of Geographically Weighted Regression With Fixed Kernel Gaussian

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Abstract

In spatial analysis, the relationship between predictor and response variables is often not uniform across regions, a condition known as spatial heterogeneity. Global regression models such as Ordinary Least Squares (OLS) assume that regression parameters are constant across all locations, meaning that the effects of independent variables are considered identical for every observational unit. This assumption is often unrealistic because it does not account for differences in regional characteristics. If spatial heterogeneity is ignored, model estimates may become less accurate and fail to explain local variations adequately. Geographically Weighted Regression (GWR) is a model that allows regression parameters to vary according to geographic location. Under this approach, each region obtains its own local parameter estimates, enabling the model to capture spatially varying relationships more flexibly. The results indicate that the AIC value of the GWR model with Fixed Gaussian Kernel (347.8637) is smaller than that of the OLS model (382.1161), suggesting that GWR provides a better fit. Empirically, the Human Development Index (HDI) in 53 districts/cities is influenced by Life Expectancy, Expected Years of Schooling, and Per Capita Expenditure, while in 66 districts/cities it is influenced by Life Expectancy, Expected Years of Schooling, Poverty Rate, and Per Capita Expenditure.

Keywords: *Fixed Kernel Gaussian, Geographically Weighted Regression, Ordinary Least Squares*

Introduction

Human Development Index (HDI) is one of the most important indicators used to evaluate the quality of human development across regions. It represents the achievement of development through three fundamental dimensions: health, education, and decent living standards. In Indonesia, HDI is widely used by government institutions, researchers, and policy makers to assess regional development performance and to identify inequalities among provinces, regencies, and cities. Although Java Island is often regarded as the economic and demographic center of Indonesia, the level of human development across its regencies and cities remains spatially uneven. Some urban areas show relatively high HDI values due to better access to education, health services, and economic opportunities, whereas several other regions still experience lower development outcomes. This condition indicates that human development is not only influenced by socioeconomic indicators but is also closely related to regional characteristics and spatial context.

In empirical studies, the relationship between HDI and its associated indicators is commonly analyzed using global regression models, particularly Ordinary Least Squares (OLS). The OLS model is useful because it provides a single set of regression coefficients that describes the average relationship between predictor variables and the response variable for the entire study area. However, this global assumption can be too restrictive when the data have spatial characteristics. In a spatial context, the effect of a predictor in one region may differ from its effect in another region due to variations in infrastructure, demographic structure, fiscal capacity, public service accessibility, and local economic conditions. Therefore, assuming that all regencies and cities share identical regression parameters may oversimplify the actual development process.

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The limitation of global regression is closely related to the concepts of spatial dependence and spatial heterogeneity. Tobler's first law of geography states that everything is related to everything else, but near things are more related than distant things (Tobler, 1970). This principle implies that neighboring regions may show similar development characteristics because they share geographical proximity, economic interaction, infrastructure networks, and social mobility. At the same time, spatial heterogeneity may occur when the relationship between variables changes from one location to another. In HDI analysis, for example, life expectancy, expected years of schooling, poverty rate, and per capita expenditure may not have the same relative contribution in all regions. Education-related indicators may be more dominant in some areas, while economic indicators or poverty may play a stronger role in others. Ignoring these spatial variations may lead to incomplete interpretation and less effective policy recommendations.

Geographically Weighted Regression (GWR) was developed to address the limitations of global regression models in the presence of spatial non-stationarity. The method was initially introduced by Brunsdon, Fotheringham, and Charlton (1996) and later systematized by Fotheringham, Brunsdon, and Charlton (2002). Unlike OLS, which estimates one global parameter for each predictor, GWR estimates local regression parameters for each observation location. The estimation is conducted by assigning spatial weights to observations based on their geographical distance from the target location. Observations located closer to a particular region receive larger weights, while observations farther away receive smaller weights. As a result, GWR enables researchers to identify how the relationship between the response variable and predictor variables varies across space.

An important component in GWR modeling is the choice of kernel function and bandwidth. The kernel function determines how spatial weights are assigned, while the bandwidth controls the degree of spatial smoothing. A fixed Gaussian kernel applies the same bandwidth across all locations and assigns weights continuously according to distance. This approach is appropriate when the study area has relatively regular spatial coverage or when the researcher aims to apply a consistent spatial smoothing structure across the entire region. In contrast, adaptive kernels allow bandwidths to vary based on the density of observation points. Previous studies have shown that the choice between fixed and adaptive kernels can affect model performance and interpretation, particularly when the spatial distribution of observations is uneven (Yacim & Boshoff, 2019; Suryowati et al., 2021).

Several studies have applied GWR to analyze socioeconomic and regional development issues. In Indonesia, GWR has been used to examine literacy activity, unemployment, poverty, maternal mortality, and regional economic indicators. Hapsery and Trishnanti (2021) applied GWR to map factors influencing reading literacy activity in Indonesia. Purnamasari and Widyaningsih (2023) compared bandwidth selection criteria in GWR using unemployment data in Java. Sunusi and Subarkah (2023) analyzed poverty using GWR with different kernel functions, while Nugroho et al. (2024) applied Mixed Geographically Weighted Regression to poverty modeling in Bengkulu Province. Suryowati et al. (2021) compared fixed and adaptive Gaussian kernel weighting in maternal mortality analysis, and Tangka et al. (2024) used adaptive Gaussian kernel weighting to model gross regional domestic product in Indonesia. These studies demonstrate that GWR is a relevant and flexible method for investigating spatially varying relationships in regional data.

However, several gaps remain in the existing literature. First, many previous GWR studies in Indonesia have focused on poverty, unemployment, literacy, mortality, and economic growth, while studies that specifically examine the spatial variation of HDI-related indicators across regencies and cities in Java Island remain limited. Second, many applied studies emphasize model comparison using goodness-of-fit statistics but provide limited discussion on how locally significant variables are distributed spatially. Third, in the context of HDI modeling, several variables such as life expectancy, expected years of schooling, and per capita expenditure are not merely external determinants but are also conceptually related to the construction of HDI.

Therefore, this study does not interpret these variables as purely causal predictors. Instead, it examines how the association and relative contribution of HDI-related dimensions vary across regions. This framing is important to avoid overclaiming causality and to provide a more accurate interpretation of the relationship among development indicators.

Based on these gaps, this study aims to analyze the spatial variation in the relationship between HDI and selected socioeconomic indicators across regencies and cities in Java Island using Geographically Weighted Regression with a fixed Gaussian kernel. The study compares the performance of the global OLS model and the local GWR model using the corrected Akaike Information Criterion (AICc). In addition, this study identifies the spatial distribution of locally significant variables affecting HDI-related patterns across regions. The novelty of this study lies in its focus on local spatial interpretation of HDI-related indicators in Java Island, rather than merely estimating a single global relationship. By mapping the regional distribution of significant variables, this study provides empirical evidence that human development policies should not be designed uniformly across all regions. Instead, policy interventions should consider local development characteristics, because the contribution of health, education, poverty, and economic indicators may differ from one region to another.

Thus, the contribution of this study is twofold. Methodologically, it demonstrates the usefulness of GWR with a fixed Gaussian kernel in capturing spatial heterogeneity in HDI-related modeling. Substantively, it provides a more localized understanding of human development patterns in Java Island, which can support more targeted and region-specific policy formulation. The findings are expected to enrich the application of spatial regression methods in regional development studies and provide an empirical basis for improving human development strategies at the regency and city levels..

Method

In this section, a brief overview of the Ordinary Least Squares (OLS) method and the Geographically Weighted Regression (GWR) approach with a fixed Gaussian kernel weighting scheme is presented, along with the procedures used to evaluate model goodness of fit. This discussion provides the conceptual foundation for understanding how spatial components are incorporated into the modeling process and how model performance is quantitatively assessed.

Ordinary Least Squares

The Ordinary Least Squares (OLS) method is a parameter estimation approach in linear regression models based on the principle of minimizing the sum of squared errors (Francis & Nwakuya, 2022). Mathematically, a linear regression model with n observations and p parameters can be expressed in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Where $\boldsymbol{Y}$ is the response variable, $\boldsymbol{X}$ is the predictor variable with dimension $(M \times N) + 1$ and $\boldsymbol{\beta}$ represents the number of parameters in the regression model with dimension $(M \times 1) + 1$.

Geographically Weighted Regression

The Geographically Weighted Regression (GWR) method is an extension of classical linear regression that allows regression parameters to vary at each observation location. Mathematically, the GWR model can be expressed in matrix form as follows (Yacim & Boshoff, 2019):

$$\boldsymbol{Y} = \boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{\beta}(u_i, v_i) + \boldsymbol{\varepsilon}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n}$$

Where

$$\boldsymbol{\beta}_{\boldsymbol{i}}(u_i, v_i) = [\beta_{1i}(u_i, v_i) \quad \beta_{2i}(u_i, v_i) \quad \dots \quad \beta_{mi}(u_i, v_i) \quad \cdots \quad \beta_{Mi}(u_i, v_i)]^T$$

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To obtain the GWR model equation, the following formulation can be used (Purnamasari & Widyaningsih, 2023).

$$\hat{\boldsymbol{\beta}}(u_i, v_i) = \left(\boldsymbol{X}^T \boldsymbol{W}(u_i, v_i)\boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{W}(u_i, v_i)\boldsymbol{Y}$$

$\boldsymbol{X}$ represents the predictor variable with dimension $(M \times N) + 1$

$$\boldsymbol{x} = \begin{bmatrix} x_{i11} & x_{i12} & \dots & x_{i1m} & \cdots & x_{i1M} \\ x_{i21} & x_{i22} & \dots & x_{i2m} & \dots & x_{i2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{in1} & x_{in2} & \dots & x_{iNm} & \dots & x_{inM} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{iN1} & x_{iN2} & \dots & x_{iNm} & \dots & x_{iNM} \end{bmatrix}$$

$\boldsymbol{Y} = [y_1 \quad y_2 \quad \dots \quad y_j \quad \cdots \quad y_n]^T$ and

$$\boldsymbol{W} = \begin{bmatrix} w_1(u,v) & 0 & \dots & 0 & \cdots & 0 \\ 0 & w_2(u,v) & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & w_n(u,v) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \dots & w_N(u,v) \end{bmatrix}$$

Fixed Gaussian Kernel

In the Geographically Weighted Regression (GWR) approach using a fixed kernel, the spatial weighting structure is determined by a constant bandwidth applied uniformly across the entire study area. This means that each location uses the same level of smoothing in estimating local parameters, even though the distances between observation points may vary. The equation for the fixed kernel is given as follows (Hapsery & Trishnanti, 2021).

$$w_{ii} = \exp\left(-\frac{1}{2}\left(\frac{d_{ii}}{h}\right)^2\right) \text{ if } d_{ij} < b$$

Where $d_{ii} = \sqrt{(u_i - u_{i^*})^2 + (v_i - v_{i^*})^2}$

Moran's I

Moran's I is a statistic used to detect spatial autocorrelation, namely the relationship or association between locations based on the values of a variable. This test is based on the concept of spatial covariance and aims to identify whether there is a clustering pattern or spatial regularity among neighboring regions (Hutapea et al., 2025). The equation used to obtain the value of Moran's I is given as follows (Fitriani et al., 2024)

$$I = \frac{\sum_{j=1}^{N}\sum_{k=1}^{N} W_{jk}\left(X_j - \bar{X}\right)(X_k - \bar{X})}{\sum_{k=1}^{n}(X_k - \bar{X})}$$

Goodness of Fit

In Geographically Weighted Regression (GWR) modeling, one of the commonly used measures to evaluate goodness of fit as well as model complexity is the Akaike Information Criterion corrected (AICc). This criterion is an extension of the Akaike Information Criterion (AIC) that has been adjusted for small sample sizes, making it more stable when the number of observations is relatively small compared to the number of estimated parameters. The AICc formula is given as follows (M. Fathurahman, 2022):

$$AICc = -2l\left(\hat{\theta}\right) + \frac{2n(P+1)}{n-p-1}$$

Where $l\left(\hat{\theta}\right)$ represents the value of the log-likelihood function for the parameter estimator.

Hypothesis Testing

Model parameter testing is conducted to determine which parameters significantly affect the dependent variable. The hypotheses are formulated as follows (Solekha & Qudratullah, 2022).

Hypotheses

$H_0: \beta_m(u_i, v_i) = 0$

$H_1: \beta_m(u_i, v_i) \neq 0; m = 1,2, \dots, M$

$$Z_m = \frac{\hat{\beta}_m(u_i, v_i)}{SE\left(\hat{\beta}_m(u_i, v_i)\right)}$$

Results and Discussion

Descriptive Analysis

This study uses five predictor variables and one response variable. These variables are presented in Table 1 below.

Table 1. Research Variables

Variable	Variable Name	Scale
Y	Human Development Index	Ratio
X_1	Life Expectancy	Ratio
X_2	Expected Years of Schooling	Ratio
X_3	Poor Population	Ratio
X_4	Expenditure per Capita	Ratio

To describe the data of the research variables, a descriptive analysis was conducted. The results can be seen in Table 2 below.

Table 2 Descriptive Analysis

Variable	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
X_1	73.6227	2.50	65.84	72.45	73.82	75.15	78.38
X_2	13.4211	1.03	11.82	12.70	13.30	13.96	17.61
X_3	111.242	72.70	3.49	68.77	102.57	144.84	446.79
X_4	14.2883	3.26	8.98	12.21	13.63	15.66	27.12

Based on Table 2, the descriptive statistics indicate that variable X_1 has a mean of 73.62 and a standard deviation of 2.50, with a median of 73.82, which is very close to the mean. For variable X_2, the mean is 13.42 with a standard deviation of 1.03. Variable X_3 has a mean of 222.24 and a standard deviation of 72.70, with a median of 102.57. Meanwhile, variable X_4, has a mean of 14.29 and a standard deviation of 3.26. Next, a correlation analysis among the predictor variables was conducted to examine the presence of multicollinearity. The visualization of the results can be seen in Figure 1.

Based on Figure 1, it can be observed that there is no high correlation among the predictor variables; therefore, it can be assumed that there is no strong correlation between the predictors. For a more detailed examination, a multicollinearity test was conducted using the Variance Inflation Factor (VIF) values for each predictor. The results are presented in Table 3 below.

Table 3 Variance Inflation Factor

Variable	X_1	X_2	X_3	X_4
VIF	2.495344	1.230905	2.084740	1.355669

Based on Table 3, it is known that the Variance Inflation Factor (VIF) values for each predictor variable < 5, therefore the multicollinearity assumption is satisfied

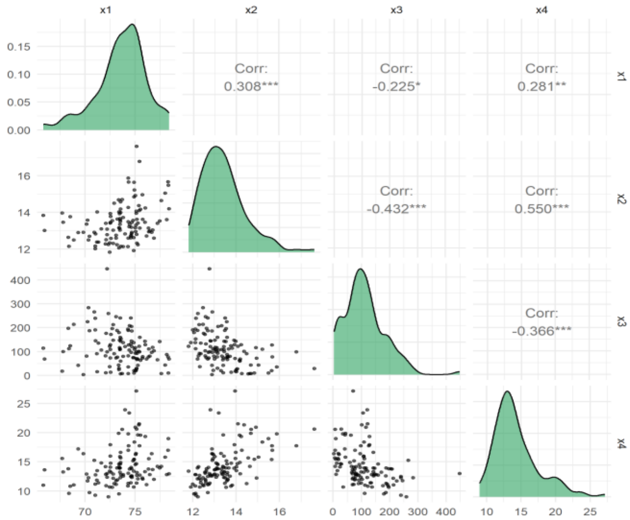

Figure 1 Analisis Korelasi

Ordinary Least Squares (OLS) Model

To describe the Ordinary Least Squares (OLS) model, a linear regression analysis was conducted. The results are presented in Table 4 below.

Table 4 Ordinary Least Squares (OLS) Model

Coefficients:	Estimate	Std.Error	t-value	p-value
(Intercept)	-10.7571	3.45582	-3.113	0.0023
X_1	0.65294	0.0458	14.258	0.0000
X_2	1.89576	0.13272	14.284	0.0000
X_3	-0.00381	0.00167	-2.281	0.0244
X_4	0.8827	0.04045	21.824	0.0000

Based on Table 4, it is known that Life Expectancy, Expected Years of Schooling, Poor Population, and Expenditure per Capita influence the Human Development Index, as evidenced by the P-Value < $\alpha_{0.05}$.

Moran's I

The Moran's I test aims to determine whether there is spatial autocorrelation in the data, that is, whether the value of a variable at one location is associated with the value of the same variable at neighboring locations. The results of the Moran's I test are presented in Table 5.

Figure 2 Analisis Korelasi

Moran I statistic standard deviate = 3.9131, p-value = 4.557e-05
alternative hypothesis: greater
sample estimates:

Moran I statistic	Expectation	Variance
0.259253331	-0.008474576	0.004681190

Based on Table 5, it is known that the Moran's I value is 0.2593, with an expected value of −0.0085 and a variance of 0.00468, with p-value = 4,557 × 10^{-5} <$\alpha_{0.05}$ Therefore, it can be concluded that spatial autocorrelation exists. Hence, further analysis needs to consider a spatial-based approach

Geographically Weighted Regression with Fixed Gaussian Kernel

In addition to examining the global model, this study also analyzes the Geographically Weighted Regression with Fixed Gaussian Kernel model. The visualization of the distribution of predictor variables that significantly influence the response variable can be seen in Figure 2 below.

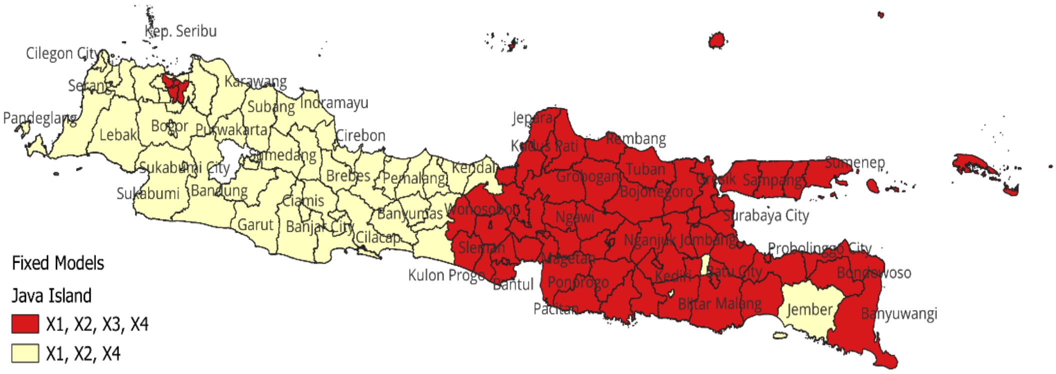

Figure 3. Geographically Weighted Regression with Fixed Gaussian Kernel

Based on Figure 2, it is known that in 53 Regencies/Cities, the Human Development Index (HDI) is influenced by Life Expectancy, Expected Years of Schooling, and Expenditure per Capita, while in 66 Regencies/Cities, the HDI is influenced by Life Expectancy, Expected Years of Schooling, Poor Population, and Expenditure per Capita. The distribution of Regencies/Cities based on significant variables can be seen in Table 6 below.

Table 5 Distribution of Regencies/Cities Based on Significant Variables

Significant Variables	Regencies/Cities				
X1, X2, X3, X4	South Jakarta	Magelang	Sragen	Jepara	Salatiga City
	East Jakarta	Boyolali	Grobogan	Demak	Semarang City
	Central Jakarta	Klaten	Blora	Semarang	Kulon Progo
	West Jakarta	Sukoharjo	Rembang	Temanggung	Bantul
	Purworejo	Wonogiri	Pati	Magelang City	Gunung Kidul
	Wonosobo	Karanganyar	Kudus	Surakarta City	Sleman
	Probolinggo	Jombang	Ngawi	Gresik	Sumenep
	Pasuruan	Nganjuk	Bojonegoro	Bangkalan	Kediri City
	Sidoarjo	Madiun	Tuban	Sampang	Malang City
	Mojokerto	Magetan	Lamongan	Pamekasan	Probolinggo City
	Kediri	Malang	Lumajang	Banyuwangi	Bondowoso
	Yogyakarta City	Pacitan	Ponorogo	Trenggalek	Tulungagung
	Pasuruan City	Mojokerto City	Madiun City	Surabaya City	Situbondo
	Blitar				
X1, X2, X4	Kep. Seribu	Cirebon	Bogor City	Banyumas	Pekalongan City
	North Jakarta	Majalengka	Sukabumi City	Purbalingga	Tegal City
	Bogor	Sumedang	Bandung City	Banjarnegara	Jember
	Sukabumi	Indramayu	Cirebon City	Kebumen	Blitar City
	Cianjur	Subang	Bekasi City	Kendal	Batu City
	Bandung	Purwakarta	Depok City	Batang	Pandeglang
	Garut	Karawang	Cimahi City	Pekalongan	Lebak
	Tasikmalaya	Bekasi	Tasikmalaya City	Pemalang	Tangerang
	Ciamis	Bandung West	Banjar City	Tegal	Serang
	Cilegon City	Serang City	South Tangerang City	Kuningan	Pangandaran
	Cilacap	Brebes	Tangerang City		

To determine the best model between the Ordinary Least Squares (OLS) model and the Geographically Weighted Regression with Fixed Gaussian Kernel model, the comparison is not based solely on Moran's I but also on the AICc values. The results can be seen in Table 7 below.

Table 6 Corrected Akaike Information Criterion

Model	AICc
Ordinary Least Squares (OLS) Model	382.1161
Geographically Weighted Regression with Fixed Gaussian Kernel	347.8637

Based on Table 7, it is known that the AICc value of the Geographically Weighted Regression with Fixed Gaussian Kernel model (347.8637) < AIC model Ordinary Least Squares (382.1161). Therefore, it can be concluded that the Geographically Weighted Regression with Fixed Gaussian Kernel model performs better than the Ordinary Least Squares model.

The comparison between the Ordinary Least Squares (OLS) model and the Geographically Weighted Regression (GWR) model with a fixed Gaussian kernel indicates that the local spatial model provides a better representation of the relationship between the Human Development Index (HDI) and its associated indicators. The lower AICc value of the GWR model shows that allowing regression coefficients to vary across locations improves model fit compared with the global OLS model. This result is consistent with the basic idea of GWR, which is designed to accommodate spatial heterogeneity by estimating location-specific parameters rather than assuming one constant relationship for the entire study area (Brunsdon et al., 1996; Fotheringham et al., 2002). The superiority of the GWR model in this study is also in line with previous empirical studies that found spatial regression models to be more informative than global models when regional data show spatial variation. Suryowati et al. (2021) showed that GWR with Gaussian kernel weighting was useful for modeling maternal mortality rates because the influence of explanatory variables varied across locations. Similarly, Sunusi and Subarkah (2023) demonstrated that the use of different kernels in GWR could improve the modeling of poverty by capturing local spatial patterns. These findings support the result of this study, where the GWR model provides a better fit than OLS in explaining HDI-related variations across regencies and cities on Java Island.

The spatial variation in significant variables found in this study also confirms that development indicators do not have uniform effects across regions. Life Expectancy, Expected Years of Schooling, and Expenditure per Capita are significant in both regional groups, indicating that health, education, and economic capacity remain central dimensions associated with HDI. However, Poor Population is significant only in some regencies and cities. This suggests that the relationship between poverty and HDI depends on local socioeconomic conditions. This result is consistent with studies by Nugroho et al. (2024) and Sunusi and Subarkah (2023), which showed that poverty-related variables may have different spatial effects across regions when analyzed using geographically weighted models.

The finding also strengthens the argument that human development policies should not be formulated uniformly for all regions. Although Java Island is generally more developed than many other parts of Indonesia, disparities among regencies and cities remain evident. Regions where Poor Population is locally significant may require stronger poverty reduction policies, improvement of household welfare, and expansion of access to basic services. Meanwhile, regions where poverty is not locally significant may require greater attention to education quality, health outcomes, and purchasing power. This interpretation is consistent with the purpose of GWR as a spatial modeling approach that provides local information for region-specific policy formulation (Yacim & Boshoff, 2019; Purnamasari & Widyaningsih, 2023).

Nevertheless, the results should be interpreted carefully because several predictor variables used in this study, namely Life Expectancy, Expected Years of Schooling, and Expenditure per Capita, are conceptually related to the construction of HDI. Therefore, these variables should not be interpreted as purely causal determinants of HDI. Rather, the results reflect spatial variation in the contribution of HDI-related dimensions across regions. This interpretation is important to avoid overclaiming causality and to position the study as an analysis of spatially varying associations among human development indicators.

Conclusion

Based on the analysis results, it is known that the AICc value of the Geographically Weighted Regression with Fixed Gaussian Kernel model (347.8637) < model Ordinary Least Squares (382.1161). Therefore, it can be concluded that the Geographically Weighted Regression with Fixed Gaussian Kernel model performs better than the Ordinary Least Squares model. In the Geographically Weighted Regression with Fixed Gaussian Kernel model, it is found that in 53 Regencies/Cities, the Human Development Index (HDI) is influenced by Life Expectancy, Expected Years of Schooling, and Expenditure per Capita, while in 66 Regencies/Cities, the HDI is influenced by Life Expectancy, Expected Years of Schooling, Poor Population, and Expenditure per Capita.

Acknowledgment

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References

Brunsdon, C., Fotheringham, A. S., & Charlton, M. E. (1996). Geographically weighted regression: A method for exploring spatial nonstationarity. *Geographical Analysis, 28*(4), 281–298. https://doi.org/10.1111/j.1538-4632.1996.tb00936.x

Fathurahman, M. (2022). Pemilihan model regresi Bernoulli terbaik. *Prosiding Seminar Nasional Matematika, Statistika, dan Aplikasinya*, 138–146.

Fitriani, R., Wardhani, N. W. S., & Abdila, N. S. (2024). Spatial autocorrelation of East Java's economic growth using cluster-based weights. *Economics Development Analysis Journal, 13*(4), 491–503.

Fotheringham, A. S., Brunsdon, C., & Charlton, M. (2002). *Geographically weighted regression: The analysis of spatially varying relationships*. John Wiley & Sons.

Francis, R. E., & Nwakuya, M. T. (2022). A comparative analysis of ordinary least squares and quantile regression estimation technique. *American Journal of Mathematical and Computer Modelling, 7*(4), 49–54. https://doi.org/10.11648/j.ajmcm.20220704.11

Hapsery, A., & Trishnanti, D. (2021). Aplikasi geographically weighted regression (GWR) untuk pemetaan faktor yang mempengaruhi indeks aktivitas literasi membaca di Indonesia. *Jurnal Riset dan Aplikasi Matematika, 5*(2), 80–91.

Hutapea, R. O., Sinaga, M. R. T., Lubis, M. S. I., & Aqil, M. F. (2025). Spatial analysis of earthquake distribution patterns in North Sumatera in 2022 using Moran's test and Moran scatter plot mapping. *EDUCTUM: Journal Research, 4*(2), 6–14.

Nugroho, S., Rini, D. S., Jomecho, T., Oktarina, C. R., Pratiwi, S. C., & Karuna, E. E. (2024). Spatial modeling of poverty in Bengkulu Province with mixed geographically weighted regression. *BAREKENG: Jurnal Ilmu Matematika dan Terapan, 18*(2), 759–772. https://doi.org/10.30598/barekengvol18iss2pp0759-0772

Purnamasari, C. P. S., & Widyaningsih, Y. (2023). Perbandingan performa bandwidth CV, AICc, dan BIC pada model geographically weighted regression: Aplikasi pada data pengangguran di Pulau Jawa. *Inferensi, 1*(1), 1–10. https://doi.org/10.12962/j27213862.v1i1.19130

Solekha, N. A., & Qudratullah, M. F. (2022). Pemodelan geographically weighted logistic regression dengan fungsi adaptive Gaussian kernel terhadap kemiskinan di Provinsi NTT. *Jambura Journal of Mathematics, 4*(1), 17–32. https://doi.org/10.34312/jjom.v4i1.11452

Sunusi, N., & Subarkah, A. (2023). Geographically weighted regression with different kernels: Application to model poverty. *Indonesian Journal of Applied Research, 4*(1), 27–41. https://doi.org/10.30997/ijar.v4i1.283

Suryowati, K., Ranggo, M. O., Bekti, R. D., Sutanta, E., & Riswanto, E. (2021). Geographically weighted regression using fixed and adaptive Gaussian kernel weighting for maternal mortality rate analysis. In *Proceedings of the International Conference on Electronics Representation and Algorithm (ICERA)* (pp. 115–120). IEEE. https://doi.org/10.1109/ICERA53111.2021.9538643

Tangka, F. E., Hatidja, D., & Deilan, W. C. W. (2024). Pemodelan geographically weighted regression dengan pembobot adaptive Gaussian kernel pada PDRB di Indonesia. *Jurnal Ilmu Sains, 24*(2), 110–119.

Tobler, W. R. (1970). A computer movie simulating urban growth in the Detroit region. *Economic Geography, 46*(Suppl.), 234–240. https://doi.org/10.2307/143141

Yacim, J. A., & Boshoff, D. G. B. (2019). A comparison of bandwidth and kernel function selection in geographically weighted regression for house valuation. *International Journal of Technology, 10*(1), 58–68. https://doi.org/10.14716/ijtech.v10i1.975